concludes from the total experimental evidence that the end conditions are of vital importance. Second, does the theory really correspond to the situation in the experiments quoted by Stein? The classical theory for the circumferential ring-type buckling,[2] does apply to either completely clamped or incompletely restrained end conditions. However, for the classical solution for the diamond buckling mode,[2] it is necessary [Ref. 2, Eq. (f), p. 464] that the diamond wrinkles extend to the ends of the cylinder, at which $u - \bar{u} = \pm A \sin n\theta \neq 0$. Therefore, the classical solution for the diamond buckling mode is not comparable with any of the experimental evidence quoted by Stein on the diamond buckling mode with completely clamped end conditions. All of his references (4–7) specifically indicate the diamond buckling mode and such severe end clamping as to completely suppress all displacements there. Furthermore, the high-speed movie-camera observations recorded in his Refs. 4 and 6 clearly show that the buckling mode observed was the diamond mode right from the onset of buckling. There is no evidence cited in any of his relevant references (4–7) of any change of buckling mode from the rings to the diamonds.

Therefore, the problem of reconciling the classical equilibrium-bifurcation analysis with the comparable experimental evidence appears to remain unsolved. Consequently, it appears to be premature to conclude that the classical shell-buckling analyses are basically correct.

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Reply by Author to L. J. Hart-Smith

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THIS reply is based on the following interpretation of certain of the expressions used by Mr. Hart-Smith (see the preceding comment). The expression "classical analysis" in the comments is interpreted to mean the same as the expression "conventional analysis" in the author's paper.[4] "Clamped" in the comments seems to refer to one type of in-surface restraint ($u = \bar{u}$, $v = 0$), whereas in the author's paper, it referred to a common type of out-of-surface restraint ($w = \partial w/\partial x = 0$). Also, evidently, "diamond buckling results" in the comments refers to conventional results based on the sine-sine configuration [given by Eq. (f), p. 464 of Ref. 2 of the comments) rather than postbuckling results corresponding to the diamond configuration that is commonly seen in experiment.

In as much as Mr. Hart-Smith's comments are directed primarily to the results presented in Fig. 1 of Ref. 4, it is appropriate to review in detail the author's interpretation of those results. Figure 1 indicates that if $v = w = 0$ at the edges, conventional theory predicts the same average buckling stress for the cylinder in axial compression no matter whether the axial displacement or the axial force is held constant at the edges or whether the edge slope or moment is zero. Consistent theory, on the other hand, indicates a variety of buckling loads depending on the edge restraint all within about 20% of the corresponding conventional theory value (see also Fig. 4, $p = 0$). All available near-perfect experimental results were plotted in Fig. 1. In all the experiments, attempts were made to obtain clamped edges ($w = \partial w/\partial x = 0$) with $u = \bar{u}$, $v = 0$; therefore, the experiments should be compared to the consistent theory for the same boundary conditions—the uppermost solid line in Fig. 1. Failure to achieve complete edge restraint would lead to lower experimental buckling loads and may contribute a small portion of the differences between experiment and this theoretical line which are shown in Fig. 1.

The equilibrium-bifurcation analysis for shell buckling has been strongly supported in the author's paper based on the agreement in Fig. 1 between theory and experiment for the buckling load. Mr. Hart-Smith questions the strong support on the basis of his interpretation of comparisons of buckling configurations obtained by experiment with that predicted by theory. This criticism deserves discussion. There has long been some misunderstanding as to what configuration should be seen at buckling. Consistent theory indicates that the configuration that should be expected from a perfect cylinder at buckling is a combination of the axisymmetric prebuckling configuration and the asymmetric initial buckling configuration (not to be confused with the final "diamond-shaped" postbuckling configuration commonly observed in the laboratory). Indication that the question may be resolved appears in Ref. 1 in which evidence has now been obtained through experiments on geometrically near-perfect cylinders that the shape at buckling is indeed a combination of an axisymmetric configuration and the asymmetric buckling configuration.

It is not clear to the author how to interpret Mr. Hart-Smith's comments with regard to "change of buckling mode from the rings to the diamonds." If the Comments refer to change from axisymmetric prebuckling deformations to asymmetric buckling configuration, then it should be remarked that these effects are accounted for in consistent theory and the prebuckling deformations have been observed experimentally.[2] If the Comments refer to change in buckle pattern with change in shell dimensions, then it should be noted that from consistent calculations made so far for the cylinder in axial compression, the critical buckling loads have always corresponded to $n \neq 0$ (see, for example, Table II of Ref. 3) and, therefore, no ring buckling has been indicated. Note that ring buckling loads for consistent theory represent asymptote values rather than bifurcation values.

Mr. Hart-Smith's statement that the classical sine-sine solution is not precisely comparable to experiment because the in-surface boundary condition on u is not satisfied is, strictly speaking, correct. However, this point is irrelevant to the author's comparison of consistent theory with experiment; moreover, the demonstrated insensitivity of both conventional and consistent solutions (Fig. 4) to change in this boundary condition suggest that comparison of even the conventional solutions with laboratory experiments should not be condemned on this basis. In summary:

1) Agreement in buckling load should be obtainable between experiment and theory for the same boundary conditions. It has been obtained for the near-perfect clamped cylinder in axial compression.

2) New evidence has been found to indicate it is possible to get the theoretical buckling configuration experimentally for this loading. The axisymmetric prebuckling configuration has been observed experimentally, but the axisymmetric buckling configuration will probably not appear in thin cylinders buckling elastically in pure axial compression.

3) These results and others cited in the author's paper for this and other loadings all point to the conclusion that equilibrium-bifurcation shell-buckling analysis is basically correct.

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Comment on "Effects of Freestream Disturbances on Boundary-Layer Transition"

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THE following comments are directed to the authors of Ref. 1. Reference 2 presents a rather comprehensive investigation of the effects of periodic disturbances of a single frequency on flat plate boundary-layer transition. In that work Miller and Fejer found that the transition Reynolds number was a function only of the disturbance amplitude, whereas the disturbance frequency dictated the transition length. Figures 1 and 2 summarize those results.

Although the minimum amplitude of disturbance investigated by Miller and Fejer was an order of magnitude greater than the maximum disturbance amplitude of Ref. 1, one would a priori expect the same phenomenological behavior at the lower amplitudes. As was predicted by Liepmann[3] in 1945, most data tend to indicate that the value of the maximum rate of shear occurring within the boundary layer dominates the transition mechanism. More recently, Greenspan and Benney[4] have confirmed this notion analytically. Thus, since one would expect the maximum instantaneous value of shear in the boundary layer to be proportional to the disturbance amplitude, one may conclude that the results reported in Ref. 2 are physically justifiable, whereas Spangler and Wells[1] offer no explanation in terms of fundamental phenomena for their rather surprising findings.

Further consideration of the experiment of Ref. 1 brings to mind that the real indicator of transition is the appearance of turbulent bursts in the late Tollmien-Schlichting flow, a phenomenon rather difficult to detect with the smoke filament technique described by Spangler and Wells. Indeed, it may well be that a careful hot-wire traverse of the transition regime will indicate results more in consonance with those previously reported. Moreover, detailed information on the distribution of the intermittency factor would be a valuable product of such an investigation.

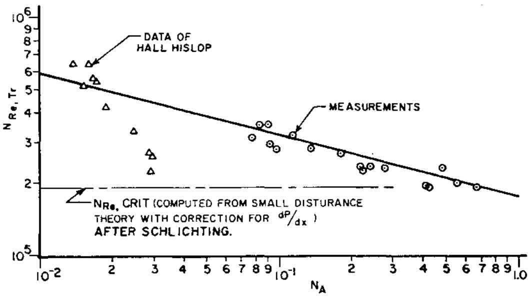

Fig. 1 Effect of the amplitude parameter $\Delta U/U_\infty$ on the transition Reynolds number.

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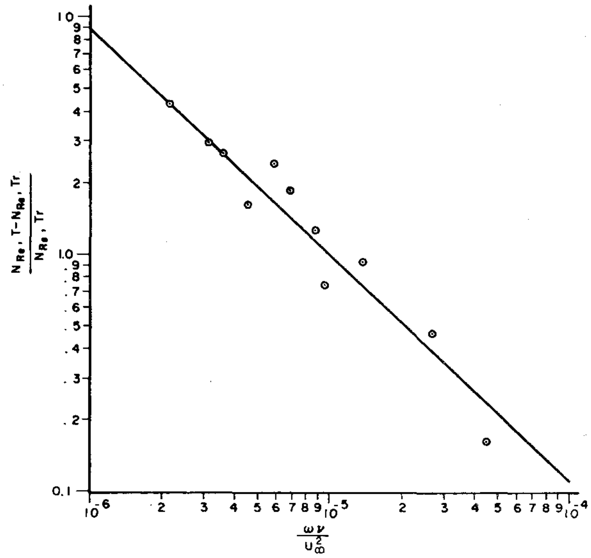

Fig. 2 Effect of the frequency parameter $\omega\nu/U_\infty{}^2$ on the transition length.

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Reply by Authors to J. A. Miller

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THE authors are familiar with the investigations of Miller and Fejer[1] concerning boundary-layer transition induced by periodic disturbances. Although on the surface the two investigations appear similar, there is one important difference that we feel explains the disparity in results. That difference is the magnitude of the freestream disturbances that induced transition. Miller and Fejer studied the effects of sinusoidal freestream disturbances ranging in amplitude from 8.0 to 67.0% of the freestream velocity, whereas our disturbances ranged from 0.04 to 0.33% of freestream, some two orders of magnitude smaller.

Transition may indeed be a function of disturbance amplitude only and not of frequency, as was found by Miller and Fejer, when the freestream disturbance is large enough to impose directly on the boundary layer a rate of shear high enough to cause breakdown. The validity of the role of maximum shear rate in the transition process has been recognized by many. Liepmann,[2] van Driest and Blumer,[3] and Rouse,[4] among others, have all considered this point. Unfortunately, no one as yet has been able to predict correctly or to measure experimentally the value of the maximum shear rate preceding transition. Consequently the minimum value

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